Trees

Tree-Structured Data

```
def tree(label, branches=[]):
    return [label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = list(branches)
```

A tree can contains other trees:

```
[5, [6, 7], 8, [[9], 10]]

(+ 5 (- 6 7) 8 (* (- 9) 10))

(S
  (NP (JJ Short) (NNS cuts))
  (VP (VBP make)
      (NP (JJ long) (NNS delays)))
  (. .))

<ul>
  <li>Midterm <b>1</b></li>
  <li>Midterm <b>2</b></li>
</ul>
```

Tree processing often involves recursive calls on subtrees

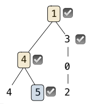
Tree Processing

Solving Tree Problems

Implement **bigs**, which takes a Tree instance t containing integer labels. It returns the number of nodes in t whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):
    """Return the number of nodes in t that are larger than all their ancestors.

    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
    >>> bigs(a)
    4
    """
```



The root label is always larger than all of its ancestors

```
if t.is_leaf():
    return ___
else:
    return ___([___ for b in t.branches])
```

Somehow increment the total count

Somehow track a list of ancestors

```
if node.label > max(ancestors):
```

Somehow track the largest ancestor

```
if node.label > max_ancestors:
```


Solving Tree Problems

Implement **bigs**, which takes a Tree instance t containing integer labels. It returns the number of nodes in t whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):
    """Return the number of nodes in t that are larger than all their ancestors.

    >>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])])
    >>> bigs(a)
    4
    """
    def f(a, x):
        if a.label > x:
            return 1 + sum([f(b, a.label) for b in a.branches])
        else:
            return sum([f(b, x) for b in a.branches])
    return f(t, t.label - 1)
```

Somehow track the largest ancestor

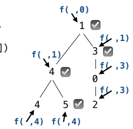
A node in t → a; max_ancestor → x

node.label > max_ancestors

Somehow increment the total count

Root label is always larger than its ancestors

Some initial value for the largest ancestor so far...



Recursive Accumulation

Solving Tree Problems

Implement **bigs**, which takes a Tree instance t containing integer labels. It returns the number of nodes in t whose labels are larger than any labels of their ancestor nodes.

```
def bigs(t):
    """Return the number of nodes in t that are larger than all their ancestors."""
    n = 0

    def f(a, x):
        nonlocal n

        if a.label > x:

            n += 1

        for b in a.branches:

            f(b, max(a.label, x))

    f(t, t.label - 1)

    return n
```

Somehow track the largest ancestor

node.label > max_ancestors

Somehow increment the total count

Root label is always larger than its ancestors

Designing Functions

How to Design Programs

From Problem Analysis to Data Definitions
Identify the information that must be represented and how it is represented in the chosen programming language. Formulate data definitions and illustrate them with examples.

Signature, Purpose Statement, Header
State what kind of data the desired function consumes and produces. Formulate a concise answer to the question *what* the function computes. Define a stub that lives up to the signature.

Functional Examples
Work through examples that illustrate the function's purpose.

Function Template
Translate the data definitions into an outline of the function.

Function Definition
Fill in the gaps in the function template. Exploit the purpose statement and the examples.

Testing
Articulate the examples as tests and ensure that the function passes all. Doing so discovers mistakes. Tests also supplement examples in that they help others read and understand the definition when the need arises—and it will arise for any serious program.

https://htdp.org/2018-01-06/Book/

Applying the Design Process

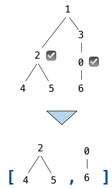
Designing a Function

Implement **smalls**, which takes a Tree instance t containing integer labels. It returns the non-leaf nodes in t whose labels are smaller than any labels of their descendant nodes.

```
def smalls(t):                  Signature: Tree -> List of Trees
    """Return the non-leaf nodes in t that are smaller than all their descendants.

    >>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])
    >>> sorted([t.label for t in smalls(a)])
    [0, 2]
    """
    result = []                    Signature: Tree -> number
    def process(t):                "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return t.label
        else:


            return min(...)
    process(t)
    return result
```

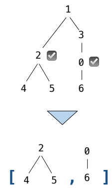


Designing a Function

Implement **smalls**, which takes a Tree instance t containing integer labels. It returns the non-leaf nodes in t whose labels are smaller than any labels of their descendant nodes.

```
def smalls(t):                  Signature: Tree -> List of Trees
    """Return the non-leaf nodes in t that are smaller than all their descendants.

    >>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])])
    >>> sorted([t.label for t in smalls(a)])
    [0, 2]
    """
    result = []                    Signature: Tree -> number
    def process(t):                "Find smallest label in t & maybe add t to result"
        if t.is_leaf():
            return ____________t.label____________
        else:
  smallest label ->  smallest = __min([process(b) for b in t.branches])__
  in a branch of t   if ____t.label < smallest____________________:
                         ____result.append( t )___________________
            return min(smallest, t.label)
    process(t)
    return result
```



Expression Trees

Interpreter Analysis

How many times does scheme_eval get called when evaluating the following expressions?

```
(define x (+ 1 2))

(define (f y) (+ x y))

(f (if (> 3 2) 4 5))
```